



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034
B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2013

MT 2501/MT 2500 - ALGEBRA, ANAL.GEO & CALCULUS - II

Date: 30/04/2013

Dept. No.

Max. : 100 Marks

Time: 9:00 - 12:00

PART - A

Answer ALL the questions

(10 X 2 = 20 Marks)

1. Evaluate $\int \frac{1}{x^2-36} dx$
2. Evaluate $\int xe^x dx$
3. Solve $(xdy + 2ydx) = xydy$
4. Define Particular Integral.
5. Show that $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$ is convergent.
6. Find the nature of the sequence $\frac{n}{n^2+7}$
7. State Raabe's test.
8. Sum the series $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.7}{6.12.8} + \dots$
9. Show that the points (5,3,-2), (3,2,1) and (-1,0,7) are collinear.
10. Find the direction cosines of the line joining the points (1, 2, 4) and (-1, 3, 2).

PART - B

Answer any FIVE questions

(5 X 8 = 40 Marks)

11. Find the length of one loop of the curve $3ay^2 = x(x-a)^2$.
12. Show that $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta) d\theta = \frac{\pi}{8} \log 2$.
13. Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$.
14. Solve $(D^2 + 4)y = x \sin x$.
15. (i) Show that $\sum \frac{1}{4n^2-1} = \frac{1}{2}$ (ii) Show that $\sum \frac{1}{n}$ is divergent. (5+3)
16. Find the image of the point $P(1,3,4)$ in the plane $2x - y + z + 3 = 0$
17. Find the equation of the plane through the points (2,2,1) and (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$.
18. Prove that $\frac{1}{2.3} + \frac{1}{4.5} + \frac{1}{6.7} + \dots = 1 - \log 2$.

PART - C

Answer any TWO questions

(2 X20 = 40 Marks)

19. Evaluate i) $\int x^4(\log x)^3 dx$.

ii) $\int \frac{dx}{x^4+1}$. (10+10)

20. a) Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$.

b) Test the convergence of the series $\frac{x}{3} + \frac{1.2 x^2}{3.5} + \frac{1.2.3 x^3}{3.5.7} + \dots$

(12+8)

21 a) Sum the series to infinity $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \dots$

b) When x is large show that $\sqrt{x^2 + 4} - \sqrt{x^2 + 1} = \frac{3}{2x} \left(1 - \frac{5}{4x^2} + \frac{21}{8x^4}\right)$ nearly. (8+12)

22 a). Find the magnitude and equations of the line of shortest distance between the lines

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

b). Find the equation to the sphere through the points

$(0,0,0), (0,1,-1), (-1,2,0)$ and $(1,2,3)$ (12+8)

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